Sri Lankan Journal of Technology
Research

SLJoT
2025 Sp.Issue 61-74
ISSN-2773-6970

Signature Authentication Using Proper Orthogonal Decomposition

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Received: 26-11-2024. * Accepted: 11-04-2025 * Published Online: 05-05-2025

Abstract- A signature is a critical tool for authenticating documents and verifying personal identity. This study introduces a novel offline signature verification approach using Reduced Order Modelling (ROM) based on Proper Orthogonal Decomposition (POD). This statistical method converts high-dimensional data into a lower-dimensional model, extracting the most important features representing the original dataset. We used 30 real signatures as training data to create the ROM and tested the model performance with 10 different signatures. The objective is to test the ROM's performance by reconstructing input signatures and verifying the test data signatures as genuine or forgeries. The required basis functions for the ROM are obtained using POD, with the eigenvalue spectrum determining the number of basis functions. Selecting 30 dominant eigenvalues, the ROM successfully reconstructed the signatures. The quality of the reconstructed signatures was evaluated using the Structural Similarity Index Measure (SSIM), yielding a similarity index value of 0.6494. The reduced matrix size was 30×2800. The Euclidean norm was used to verify signatures, and the best confidence interval was determined from four significance levels, with the 99.9% level providing the best verification results and 64% accuracy. Future work will focus on enhancing classification accuracy by implementing new classification techniques and increasing both the testing and training sample sizes.

Keywords: Accuracy, Dimension reduction, Euclidean norm, Proper orthogonal decomposition, Reduced order model, Signature verification

Subawickrama, H. D. A. W., Nishantha, S. A. A., Udagedara, U. G. I. G. K., & Mohamed, M. A. M. (2025). Signature authentication using proper orthogonal decomposition. *Sri Lankan Journal of Agriculture and Technology. 6* (Special Issue), 61-74.

1. Introduction

A. Signature Verification

A signature is a unique key to use in documents and personal authentication. Especially, in the fields such as banking, insurance, document management, etc. The verification process aims to determine if a questioned document and a known document share the same author. The signature may change due to factors like age, mood, and environment can cause variations in a person's signature. A Signature Verification System (SVS) addresses these challenges, with two main types: offline and online verification. Online signature verification systems use a special pen called a stylus to create a signature and consider the different dynamic information such as pen location, speed, and pressure. However, the Offline signature verification systems used statistical information of signature images attainable by a scanner or a digital camera. This paper focuses on the offline system only. Training and testing are the main two stages used in the signature verification system. In the training stage, a set of genuine signature samples is used for preprocessing and feature extraction and then put into the classifier to obtain the model. In the test stage, a personal model to discriminate among writers and all types of forgeries is used and the signatures are put into the classifier for comparison and output verification results. Random, simple, and skilled forgeries are the three classes of forgeries. Random forgery is written by a person who doesn't know the shape of the original signature and who is not necessarily enrolled in the signature verification system. Simple forgery, which is represented by a signature sample, is written by a person who knows the shape of the original signature without much practice. The skilled forgery is represented by a suitable imitation of the genuine signature model.

B. ***Proper Orthogonal Decomposition***

In this study, an efficient offline signature verification method was proposed based on Proper Orthogonal Decomposition. This method is ideal for quickly authenticating large volumes of documents, such as bank cheques, where manual verification is impractical. The POD was first developed by Kosambi (1943) and this method was suggested independently by several, scientists Love (1945), Karhunen (1946), Pougachev (1953), and Obukhov (1954). In other disciplines, the same procedure goes by the name Karhunen-Love decomposition or principal component analysis (PCA). POD is a data reduction tool that generates optimal basis functions to represent a system's energy or dynamics through snapshots of the system's state over time. These snapshots form POD-based functions that allow the system's state to be reconstructed with minimal error. The process involves using an orthogonal transformation matrix based on the eigenvectors of the sample covariance matrix to de-correlate variables. The data are then projected onto a subspace defined by the eigenvectors corresponding to the largest eigenvalues, which capture the most significant features of the signature. As an example, if we have high dimensional data, maybe we have megapixel images or videos with high resolution, this POD helps us to reduce the data into the key features necessary for analyzing, understanding, and describing the data.

C. Objectives and Used Techniques

The main objective of this study is to test the performance of the reduced order model (ROM) by reconstructing the input image and verifying the signatures as genuine or forgeries. The signature verification system can be decomposed into three stages: data acquisition and preprocessing, feature extraction, and verification. Data acquisition is the process of sampling signals that measure real physical conditions and converting the resulting samples into digital values that can be manipulated by a computer. Preprocessing is an important stage in image processing. It helps us to reduce noise interference and improve the accuracy of feature extraction and verification. Binarization, filtering, and resizing are the main steps of this stage. In image processing, feature extraction is a special form of dimensionality reduction. When the input data to an algorithm is too large to be processed and it is suspected to be redundant, it will be transformed into a simplified representation set of feature vectors by carefully choosing relevant information from the input data. The Low quantity of available signature samples versus the high number of extracted features is one of the problems researchers come across in offline signature verification. Luana Batist (2007) have mentioned some remedies for that issue; they are selecting the most discriminating features, using regularization techniques to obtain a stable estimation of the covariance matrix, generating synthetic samples, and using dissimilarity representation. So, this proposed method helps us to reduce those issues by using model reduction. The ROM's required basis functions are obtained using the proper orthogonal decomposition. The eigenvalue spectrum is used to get the required number of basis functions. We showed that the ROM can successfully reconstruct the signatures. The quality of the signature is tested by the Structural Similarity Index Measure (SSIM). Lastly, the Euclidean norm (distance) is used to do the verification process. It is one of the most favorite methods for measuring the distance between vectors. The performance quality is measured by error rates such as True Positive Rate (TPR), and False Positive Rate (FPR) and measures the model accuracy for 4 different significant levels.

2. Literature review

In this section, we listed and analyzed related works in data acquisition and preprocessing feature extraction and verification from various offline systems.

A. Data Acquisition and Preprocessing

Data acquisition for online and offline systems is different. On-line signature verification systems use a special pen called a stylus, hand gloves, special tablets, a personal digital assistant (PDA), and a tracking camera to create a signature and consider the different dynamic information such as pen location, speed, pressure, stroke order, and direction, etc. However, the Offline signature verification systems used statistical information on signature images attainable by a scanner or a digital camera. Preprocessing is an important step in the signature verification system, especially in an offline system. Because the signatures are captured by using a scanner and it contains a lot of noise. The purpose of this step is to prepare a standard image for the feature extraction stage. There are more preprocessing methods found for offline systems in previous research. The most popular methods used is filtering and noise removal. Two commonly implemented filters, used for noise removal are the average filter and the median filter. Under the median filter, each pixel in the image is considered and first neighboring pixels are sorted and the original values of the pixel are replaced by the median of the list. The averaging filter replaces each pixel with its average pixel value of it and a neighborhood window of adjacent pixels. The effect of these two filtering methods is to give a smoother image with sharp features removed. Furthermore, resizing is used to remove noises and improve the accuracy of feature extraction and verification.

B. Feature Extraction and Verification

A powerful feature extraction module must be equipped with feature reduction techniques where the module is capable of extracting the most salient features from a large set of features. The current research status of signature verification feature extraction algorithms mainly extracting signature texture features, geometric features, and dynamic features. Faiza proposed an automatic recognition technology based on multi-level feature fusion and optimal feature selection and calculated 22 GrayLevel Co-Occurrence Matrix (GLCM) features and 8 geometric features, geometric features were used to characterize the shape of the signature

such as edge, and area. Bhunia proposed a signature verification method that relied on the author by using two different types of texture features, discrete wavelet features and Local Quantized Patterns (LQP) features, extracting two types of transformation based on the signature image. The signature verification methods are divided into two main classes. They are model-based verification and distance-based verification. In model-based verification, methods describe data distribution by generating models.
Hidden Markov Model (HMM) (Justino 2000), Neural Network, and Support Vector Machine (SVM) are examples of that method. In distance-based method mainly used distance measures to compare the test signature with the reference signatures. Dynamic Time Warping (DTW) (Yoshimura and Yoshimura, 1997) is an example of that method. Also, directional pdf (Drouhard, 1996), stroke extraction (Lau, 2002), synthetic discriminant functions (Wilkinson, 1991), granulometric size distributions (Sabourin, 1997), grid features (Qi and Hunt, 1994), and elastic matching (Bruyne and Forre, 1986) are different methods used for signature verification. Charu Jain, Priti Singh, and Preeti Rana (2013) proposed an offline signature verification system based on Gaussian Mixture Models (GMM). Computation of GMM-based loglikelihood probability match score, mapping of this score into soft boundary ranges of acceptance or rejection through the use of z-score analysis and normalization function, arrive the final decision of accepting or rejecting a given signature sample by using threshold are the three main layers of statistical techniques used in the verification phase. Aini Najwa Azmi, Dewi Nasien, and Fakhurul Syakirian Omar (2016) proposed a technique for SVS that uses Freeman chain code (FCC) as data representation and verification utilized Euclidean distance to measure and match in k-Nearest Neighbors. Most of the new approaches are done by combining the above two methods or modifying the old method by adding new information or new features.

Here, I include a few of those approaches. A. Piyush Shanker and A.N. Rajagopalan (2007) proposed a new offline signature verification method by modifying Yoshimura and Yoshimura's DTW algorithm to account for the stability of various sections of a signature to match suitably derived 1-D features extracted from digitized images of signature. Meenakshi K. Kalera (2004) proposed a method that used Gradient, Structural, and Concavity (GSC) features for feature extraction and a correlation-based similarity measure used for verification. Kumar (2012) proposed a method in which a novel set of features based on the surrounding property of a signature image and verification is based on Multilayer Perceptron (MLP) and Support Vector Machine (SVM). Yiwen Zhou, Jianbin Zheng, Huacheng Hu, and Yizhen Wang (2021) proposed a new method which is a combination of online and offline. They used SVM to process offline signature images and DTW to process online signature images.

3. Methodology

The person's real signature will change with mood, time, age, and other factors. As well as the forger will also imitable the signature with a lot of training in advance. So, it is necessary to extract and select more comprehensive and representative signature features. This proposed method involves the calculation of the eigenvalue decomposition of a data covariance matrix. And transform the sample covariance matrix into the basis of the eigenvectors performed. This method is an order reduction tool by projecting high-dimensional data into a lower-dimensional space which extracts the most important features that represent the more characteristic features of the original data set.

The proposed method described in this paper is implemented using MATLAB. MATLAB has a powerful collection of computational algorithms and mathematical functions like matrix inverse and matrix eigenvalues as it allows easy matrix manipulation. So, Figure 01 shows the research framework of our proposed verification system.

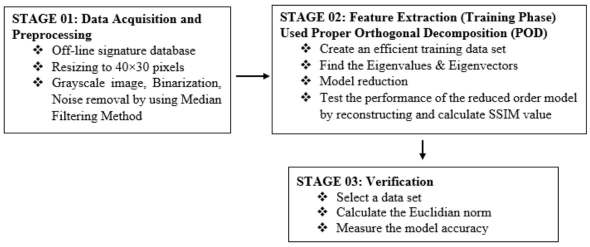

Figure 1. *Research framework*

A. Methodology for the first Two stages

1). STAGE 01: Data Acquisition and Preprocessing

An offline signature database has been used to follow the steps for verification. Here, we have taken 30 different signatures saved in .png format of five subjects with different sizes of pixels as our training database set. We use a test database set, containing 10 different signatures and each signature has 24 genuine and 4 forgeries. Under the preprocessing step, convert the original image into a grayscale image, and binarization was done to every signature image. After that, the median filter was applied to remove image noises. Finally, resized the median filter signature images into a standard size of 40 x70 pixels. So, these steps helped us to prepare a standard image for the feature extraction stage with a smoother image and improve the accuracy of feature extraction and verification.

2). STAGE 02: Feature Extraction (Training Phase)

In this stage, we tried to complete the feature extraction process. To create an efficient training data set, finding the eigenvectors and eigenvalues, model reduction, reconstruction, and measuring the SSIM values are included. The covariance matrix (S) is used to find the eigenvalues and eigenvectors.
The covariance matrix of the rows of X (X = vectorize image matrix)

$$\mathrm{n} = \text{number of images}$$

$$\mathrm{S} = \frac{\mathrm{XX^T}}{\mathrm{n}} \qquad (1)$$

The size of the matrix S is 2800×2800, which is very large and this matrix is for a data set of an old axis system where data is correlated. The required basis functions for the reduced order model are obtained using the proper orthogonal decomposition. The eigenvalue spectrum is used to obtain the required number of basis functions that truncate at the elbow point. Then we select only the L number of eigenvectors corresponding to the L numbers of dominant eigenvalues which extracts the most important characteristic features from the training signatures. Therefore, the new size of the reduced matrix is L× 2800. So, we completed the whole training phase and at the same time, the performance of the reduced order model was tested by reconstructing an input image.

The process of transforming the acquired raw data into images is called image reconstruction. The main goal of the reconstruction was to assess the effectiveness of the proposed approach, which enables to reduce of the input data space by determining the independent POD coefficient (L). If we can reconstruct the input image without loss of accuracy with small errors, this confirms that the algorithm is effective. Here we have reconstructed one of the input images for five different eigenvalues.

Structural Similarity Index Measure (SSIM) is used to measure the quality of the reconstruction image by comparing the input image. Structural Similarity Index Measure (SSIM) quantifies the image quality degradation caused by processing such as data compression or by losses in data transmission. It can perceive the changes in structural information of the image by comparing the local region of the image instead of globally. The parameters of the SSIM equation include the (x, y) location of the N×N window in each image,

The mean of the pixel intensities in the x and y direction (Luminance)

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i, \qquad \mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{2}$$

The variance of intensities in the x and y direction, along with the covariance.

$$\sigma_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)^2,$$
$$\sigma_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \mu_y)^2 \tag{3}$$

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{4}$$

$$C_1 = (k_1 L)^2 \quad C_2 = (k_2 L)^2 \tag{5}$$
$$k_1 = 0.01 \qquad k_2 = 0.03$$
$$L = \text{Dynamic range of the pixel values}$$

That value varies between -1 and 1. If that SSIM value is closer to 1 indicates a higher similarity (quality). Then we plotted the SSIM Vs eigenvalue plot and measured the similarity index values of the reconstructed images.

A. Methodology for the verification process

Finally, in Stage B, verification is the process of testing whether a signature is from the same person or not. In our case, as a total, we have trained 180 signatures which include 18 genuine signatures from each 10 different signature types. To get rage for the verification, we have used another 40 signatures which include 4 genuine from each type. For the testing phase, we have used 60 signatures which include 4 forgeries and 2 genuine ones from each signature class.

Euclidean norm (distance) is one of the most favorite methods for measuring the distance between vectors. It is equal to the square root of the matrix trace of AA^T. The trace of an n×n square matrix A is defined to be the sum of the diagonal elements.

$$\mathrm{Tr}(A) = \sum_{i=1}^{n} x_{ii} \tag{6}$$

So,

$$\text{Euclidean norm} = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}^2} = \sqrt{\mathrm{Tr}(AA^T)} \tag{7}$$

[11] The eigenvalue decomposition trace of AA^T can be represented as the summation of eigenvalues,

$$\mathrm{Tr}(AA^T) = \sum_{i=1}^{\min(m,n)} \sigma_i^2 \tag{8}$$

So,

$$\text{Euclidean norm} = \sqrt{\sum_{i=1}^{m}\sum_{j=i}^{n} A_{ij}^2}$$

$$= \sqrt{\mathrm{Tr}(AA^{T})} = \sqrt{\sum_{i=1}^{\min(m,n)} \sigma_i^2} \quad (9)$$

Then we can write the Euclidean norm as,

$$\text{Euclidean norm} = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2} \quad (10)$$

Here, σ_i^n, are the eigenvalues of AA^T

To do the verification process we have followed the main steps below mentioned.
Step 01: First of all, we trained our 180 data set which includes all preprocessing steps. Calculate the covariance matrix and find the eigenvalue matrix which represents the diagonal values. Plot the eigenvalue spectrum and choose the region of the dominant eigenvalue.

Step 02: After that replace a new genuine signature from the above-mentioned 40 data set for the 180th place into the data matrix. Again, the covariance matrix is calculated, and the eigenvalues are found. Using that calculate the Euclidean norm. Then repeat this process up to the 40th genuine signature which is mentioned above. So, we have taken 40 Euclidean norms. Using those norms, we measured the confidence interval to do the verification process at 4 different significant levels (90%, 95%, 99%, and 99.9%).
The general equation for the confidence interval (CI)

$$CI = \bar{x} \pm Z_c\left(\frac{\sigma}{\sqrt{n}}\right) \quad (11)$$

$\bar{x}$ = Sample mean
Z_c = Z Value for confidence level
n = Sample size
σ = Sample standard deviation

Step 03: Then we started to do the testing phase using the 60-signature data set that I mentioned above. Then we follow the above step 02 again up to finding the Euclidean norm. So, if this distance (norm) is in the defined range the test signature is verified to be that of the claimed subject else detected as a forgery.

To measure the performance quality, we have used two types of error rates True Positive Rate (TPR), and False Positive Rate (FPR) as well as we have measured the model accuracy for above mentioning significant levels. True Positive Rate (TPR) is the probability of positive test results, conditioned on the individual truly being positive. The false Positive Rate (FPR) is the proportion of negative cases incorrectly identified as positive cases in the data. And the accuracy measures the percentage of all correctly classified observations. To get the best performance results TPR should be high and FPR should be small.

We have used the bellow equations to measure the above-mentioned error rates and the accuracy.

$$TPR = \frac{TP}{TP+FN} \quad (12)$$

$$FPR = \frac{FP}{FP+TN} \quad (13)$$

$$\text{Accuracy} = \frac{TP+TN}{TP+TN+FP+FN} \quad (14)$$

True Positive (TP) = A test result that correctly indicates the presence of a condition.
True Negative (TN) = A test result that correctly indicates the absence of a condition.
False Positive (FP) = A test result that wrongly indicates that a particular condition is present.
False Negative (FN) = A test result that wrongly indicates that a particular condition is absent.

So, we have measured the model accuracy for 90%, 95%, 99%, and 99.9% significant levels. And we have selected the best significant level which gives higher TP and TN values as well as better accuracy values. For the easiness, here I have plotted the summary table to get the verification results.

Table 1

Summary table for the verification process.

	Genuine	**Forgery**
Genuine	TP	FP
Forgery	FN	TN

True Positive (TP) = A genuine signature correctly verified as genuine.
True Negative (TN) = A forgery signature correctly verified as a forgery.
False Positive (FP) = A forgery signature incorrectly verified as genuine.
False Negative (FN) = A genuine signature incorrectly verified as a forgery

4. Results and discussion

*A. **Results for the preprocessing and the feature extraction process***

This section presents the results of the proposed signature verification technique by using proper orthogonal decomposition. The proposed method described in this paper is implemented using MATLAB.

After doing all preprocessing steps it gives the standard images for the feature extraction stage with more smooth images with a standard size of 40 x70 pixels. So, Figure 02 shows the output of our training data set after doing all preprocessing.

Figure 2. Training dataset after completing the first stage

After calculating the data covariance matrix (S), it gives the 2800×2800 matrix size, which is very large and this matrix is for a data set of an old axis system where data is correlated. The eigenvectors obtained from the S are also the same size as the covariance matrix (S).

Here we consider the L number of eigenvectors corresponding to the L numbers of dominant eigenvalues which extracts the most important characteristic features from the training signatures. Thus, the reduced order model consists of only L number of eigenvectors. To achieve that purpose, we have used the eigenvalue spectrum shown in Figure 03, which only considers the first 50 eigenvalues (L). When L=10, it starts to perform reconstruct the input images. As mentioned in the methodology section, reconstructing the input image without loss of accuracy with a small error confirms that the algorithm is effective.

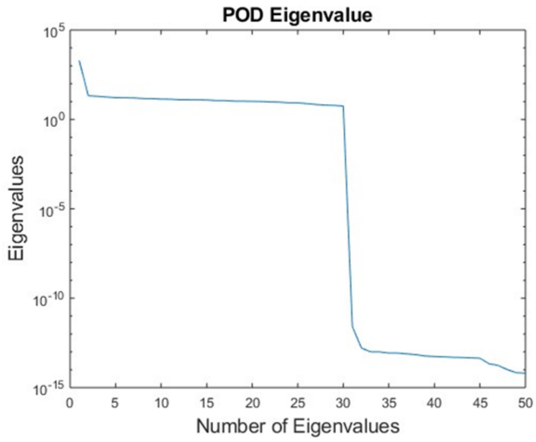

Figure 3. *Eigenvalue spectrum*

So, here I have compared my reconstructed results for 5 different eigenvalues (L=10, 15, 20, 25, 30) and measured their SSIM values. Below Figure 04 shows the reconstructed images for that eigenvalues.

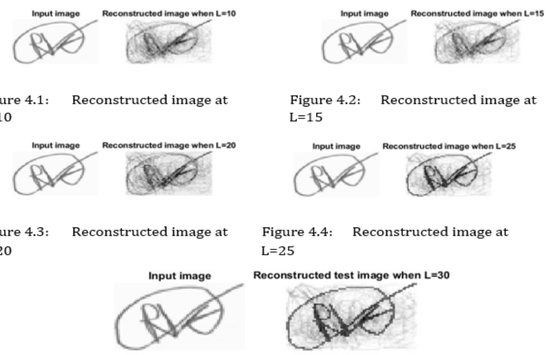

Figure 4.1: Reconstructed image at L=10

Figure 4.2: Reconstructed image at L=15

Figure 4.3: Reconstructed image at L=20

Figure 4.4: Reconstructed image at L=25

Figure 4.5: Reconstructed image at L=30

Figure 4. *Reconstructed images at L=10, 15,20,25,30*

Compared with the above-reconstructed images, we can see that when L=25 and 30 it gives the best-reconstructed images. To verify further, we used the SSIM Vs Dominant Eigenvalue plot to check the quality of the reconstructing image, which is shown in Figure 5. The structural Similarity Index (SSIM) value measures the perceptual difference between similar images. That value varies between -1 and 1. If that SSIM value is closer to 1 indicates a higher similarity (quality). Because of that we always try to reconstruct the input image with a better SSIM value which is near 1.

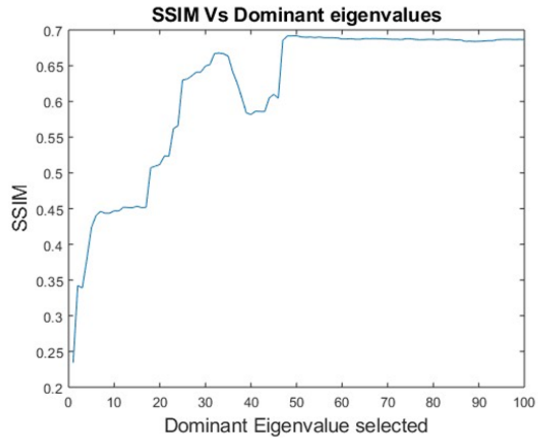

Figure 5. *SSIM Vs Dominant eigenvalue plot*

So, this plot shows that when we select the number of dominant eigenvalues as 25 or 30, it gives the quality image the same as the input image with a better SSIM value which is near 1. Using this plot, you can select the dominant eigenvalue as 40 or 50 or any value greater than 30. But here we always try to get the smallest L number of eigenvalues which gives accurate results as much as possible to get the effective reduce order model. Here I have extracted the SSIM values for the above-reconstructed images for further clarification.

Table 2
SSIM values for above-reconstructed images.

Number of Eigenvalues(L)	SSIM value
10	0.4469
15	0.4533
20	0.5118
25	0.6301
30	0.6494

According to the above table, we can confirm that when L=25 and L=30 it gives the best similarity index value for the reconstructed image. So, here I have achieved one of the objectives, which is to test the effectiveness of the proposed approach, which enables it possible to reduce the input data space by determining the independent POD coefficient (L). According to that, we can get the new size of the reduced matrix as 25×2800 or 30×2800.

B. Results for the Verification Process

Here, the probability of the signature distance being accepted as genuine or a forgery is determined. As I mentioned in stage 3, we have done the verification process using the three mentioned steps. Bellow table 03 shows the confidence interval for the four different significant levels.

Table 3
Confidence intervals for the four different significant levels.

Significant Levels (%)	**Confidence Intervals ($x_1 < a < x_2$)**
90	$48.9838 < a < 48.9903$
95	$48.9831 < a < 48.9909$
99	$48.9818 < a < 48.9922$
99.9	$48.9802 < a < 48.9939$

So, if the testing results are in the defined range the test signature is verified to be that of the claimed subject or else detected as a forgery.

We have gotten the following outcomes for the mentioned interpretations of Table 01 in the methodology section at four different significant levels while considering the confidence intervals mentioned in Table 03.

Table 4
Testing results using 40 forgery signatures at four different significant

Significant Levels (%)	**True Negative (TN)**	**False Positive (FP)**
90	32	8
95	31	9
99	28	12
99.9	26	14

Also, according to the above results shown in Table 04, at a 99.9% significant level, it gives a higher value for TP. So, it means that at the 99.9% significant level, we can verify more genuine signatures than the other significant levels.

Then we calculate the True Positive Rate (TPR) and False Positive Rate (FPR) using the above 12 and 13 equations. So, these two error rates are used to measure the performance quality of the model. To get the best performance quality it should give a higher value for TPR and a smaller value for the FPR.

Table 5
True Positive Rate (TPR) values and False Positive Rate (FPR) values.

Significant Levels (%)	**True Positive Rate (TPR)**	**False Positive Rate (FPR)**
90	0.35	0.2
95	0.35	0.225
99	0.45	0.3
99.9	0.6	0.35

So, compared with the results in Table 05, we can see that at the 99.9% significant level, it gives the best results for TPR and FPR than the other significant level.

The below table shows the accuracy results for the four different significant levels.

Table 6
Accuracy values at four different significant levels.

Significant Levels (%)	Accuracy (%)
90	65
95	63.333
99	61.667
99.9	64

Here, I have used equation 14 to measure the model's accuracy. So, accuracy measures the percentage of all correctly classified observations. Compared with the above results, at a 90% significant level, it gives the best accuracy values than the other.

C. Comparison Analysis

When using the above results in section B, I have done a comparison to select the best model that gives better performance quality.

Table 7
Comparison table using above results.

Significant Level (%)	90%	99.9%
True Negative (TN)	32	26
True Positives (TP)	7	12
True Positive Rate (TPR)	0.35	0.6
False Positive Rate (FPR)	0.2	0.35
Accuracy (%)	65	64

At the 90% level, it verified 32 forgeries from 40 forgery signatures that we tested but only 7 genuine from 20 genuine signatures that we used. At the 99.9% level, it verified 26 forgeries from 40 forgery signatures and 12 genuine from 20 genuine signatures we used. And it gives better results for the TPR than the 90% level. If we compare the accuracy values, at a 90% significant level it gives the best accuracy than the 99.9% level. However, a good verification process should verify the forgery and genuine signatures as much as possible. Because of that the verification process is done as best as possible at the 99.9% significant level. Because at that level it gives better verification results than the other level.

5. Conclusion

The required basis functions for the reduced order model are obtained using the proper orthogonal decomposition. The eigenvalue spectrum is used to obtain the required number of basis functions for the reduced-order model. When we select the number of dominant eigenvalues as 30, the reduced order model can successfully reconstruct the signatures.

Structural Similarity Index Measure (SSIM) also implies the quality of the reconstructed signatures. It gives the 0.6494 similarity index value. So, it confirms that the proposed algorithm is an effective one to use in signature verification. And we can get the new size of the reduced matrix as 30× 2800. Using the concept of the Euclidean norm, we have verified whether the signatures as genuine or not. To do that, we have got the best confidence interval from 4 different significance levels. And measure the True Positive Rates (TPR) and False Positive Rates (FPR). So, at the 99.9% significance level, it gives the best verification results as well as 64% accuracy.

Future works

Future work will aim to boost classification accuracy by adopting advanced techniques and increasing both testing and training sample sizes. We will also incorporate varied signature datasets and enhance feature extraction methods to improve model robustness and accuracy in distinguishing genuine signatures from forgeries.

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